Singular Foliations for Knit Graph Design

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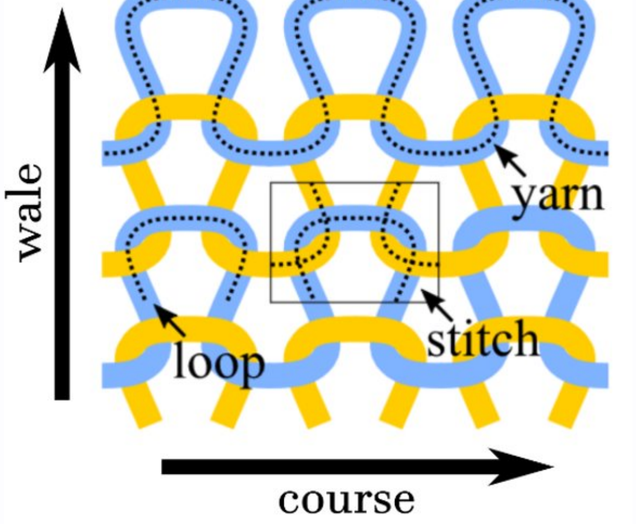
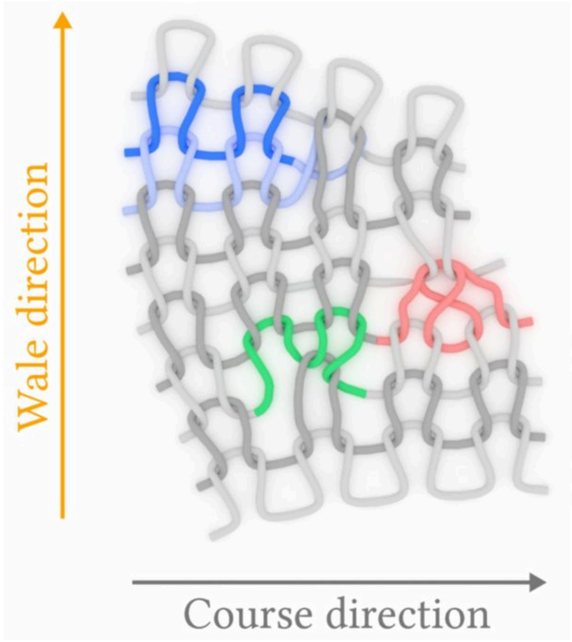
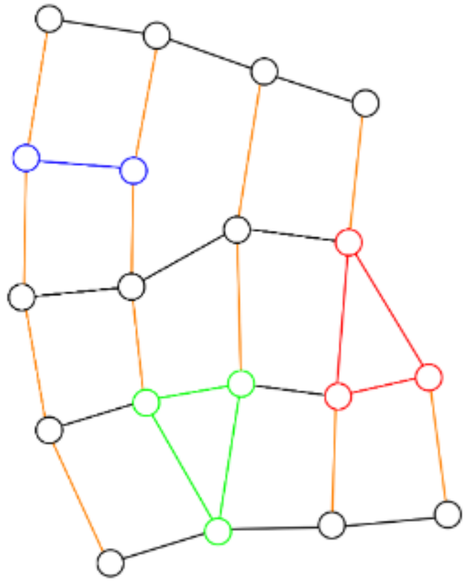
Knit Graph Abstraction

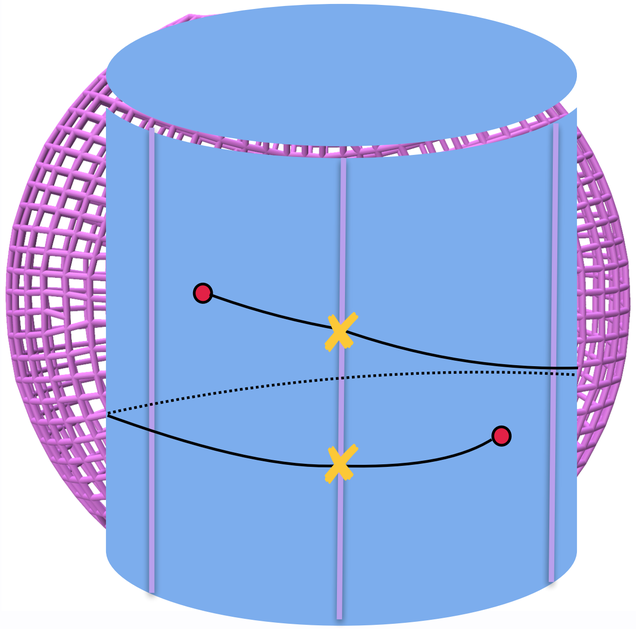


Flat geometry[1]





Stitch irregularities induce curvature[2]



Goal: Given a 3D mesh, generate **Helix-free** knit graph over it

[1] *from Visual Knitting Machine Programming (2019)*
[2] *from Knit Sketching: from Cut and Sew Patterns to Machine-Knit Garments (2021)*

Positioning our work

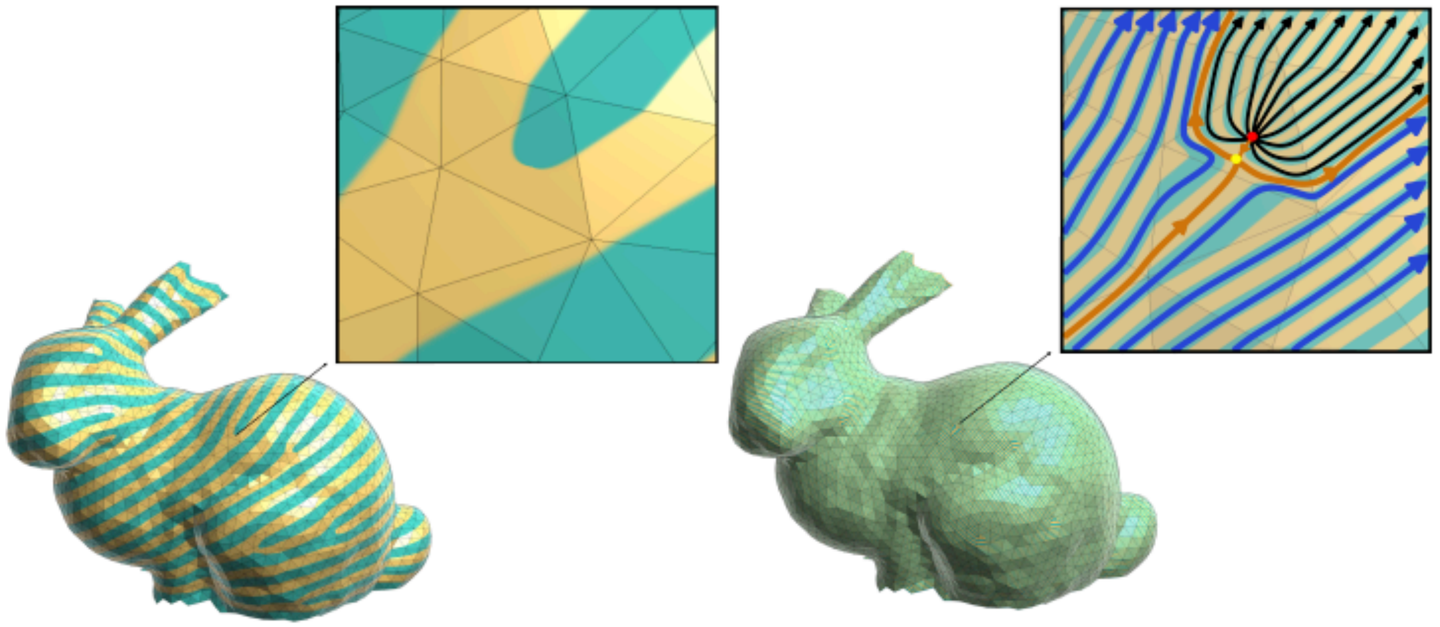
- **Goal**: Input 3D model ⟶ whole garment machine-knittable graphs
  - Similar works: **Autoknit** [Naryanan et al. 2018], **KnitKit** [Nader et al. 2021], **Helix-free Stripes** [Mitra et al. 2023]

- Stripe patterns for knitting
  - Evenly-spaced stripes ⟷ evenly spaced course rows and wale columns
  - **KnitKit** [Nader et al. 2021] - Quad mesh operators
  - **Helix-free Stripes** [Mitra et al. 2023] - Several linear constraints for helix-removal

Knöppel et al. [2015]

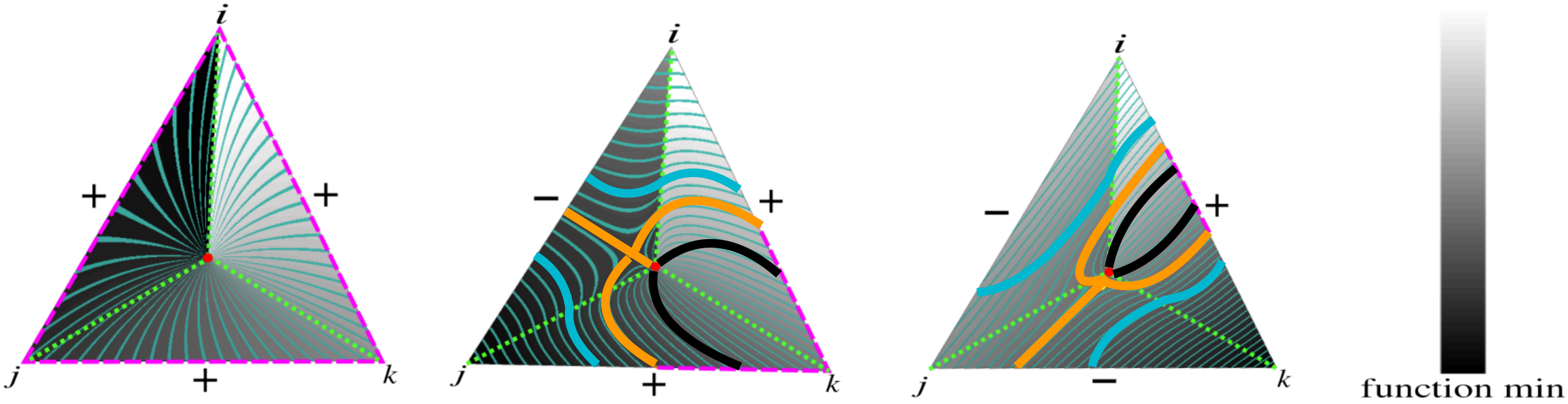
Extensions to our previous work

- Stripe patterns as **oriented foliations (integral curves of vector field)**
  - Can remove helicing of **any stripe integral curve**
  - Previous work can remove helicing in only certain integral curves
- Automatic matching of singular triangles
  - Minimum weight matching problem
  - Customized weights



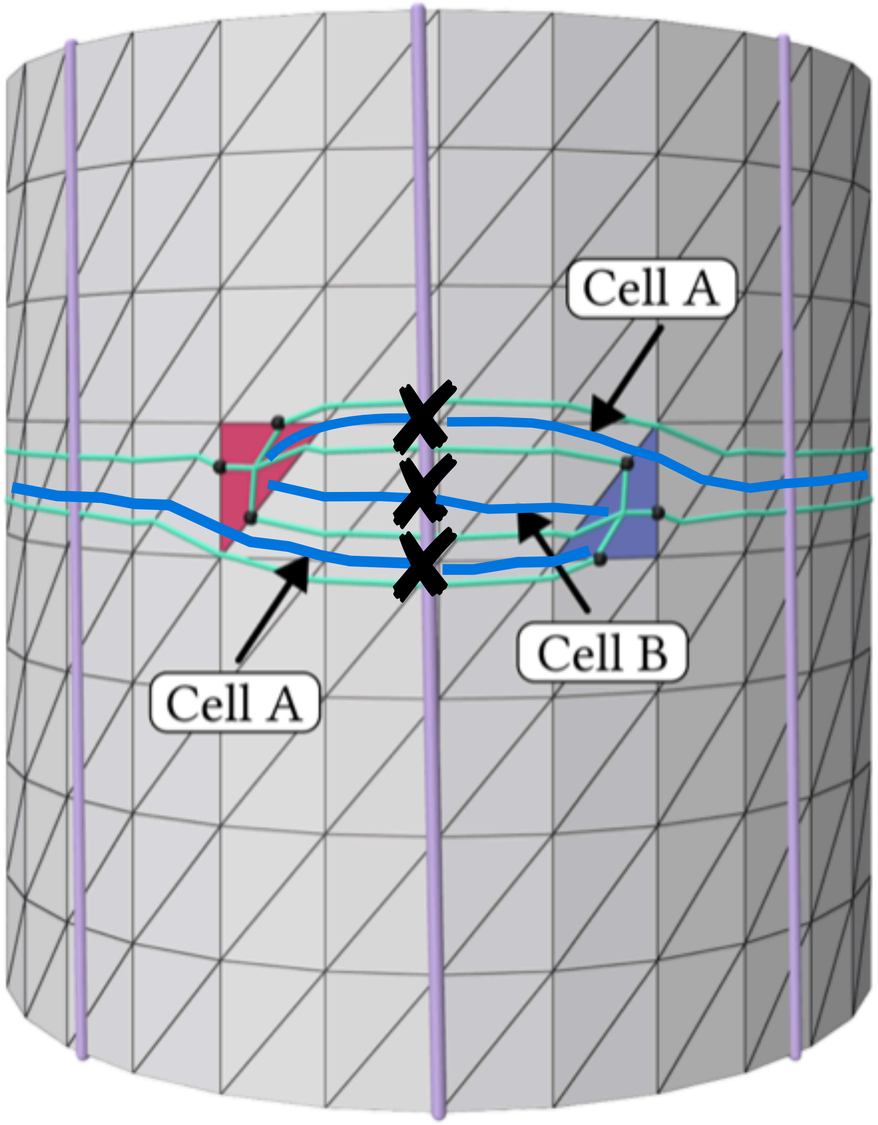
Foliation Behavior over Singular Triangles

Knöppel et al. [2015] stripes on singular triangles

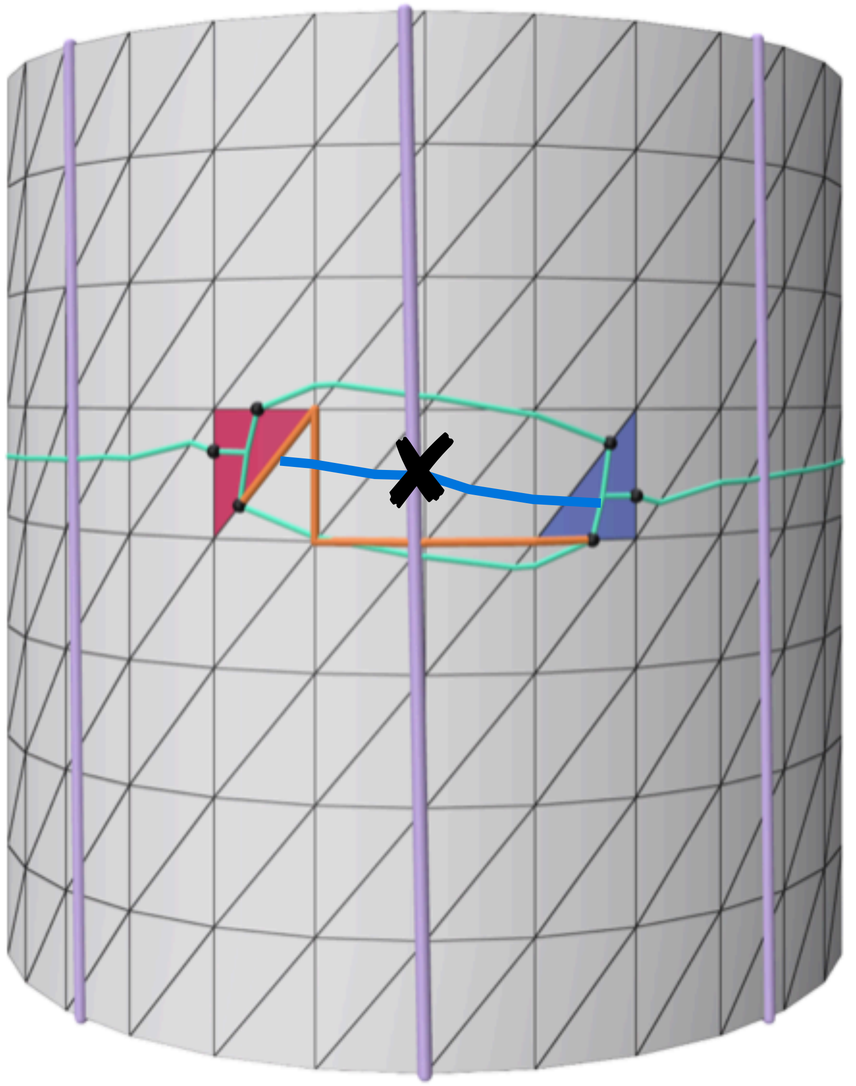


- Upshot: Exact **characterization** of where the **separatrices** hit the boundary
  - Matching separatrices enforces helix-free condition

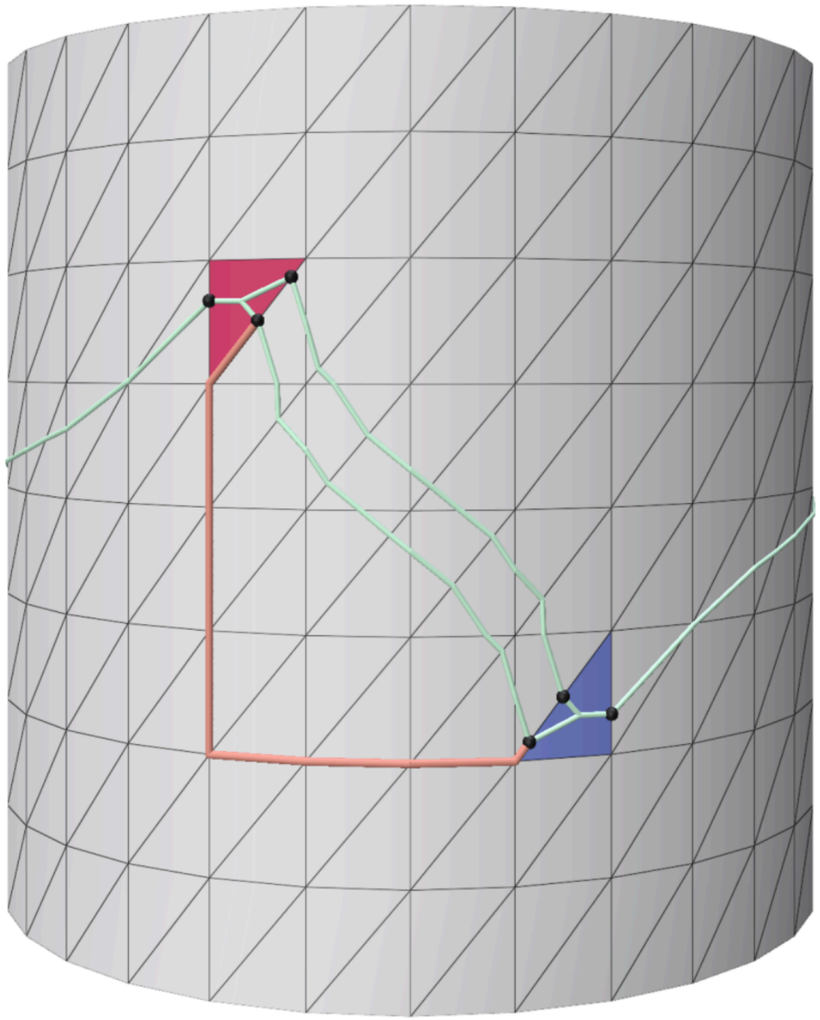
Precise Topological Control of Stripes



Automatic Separatrix Matching



Automatic Singularity Matching



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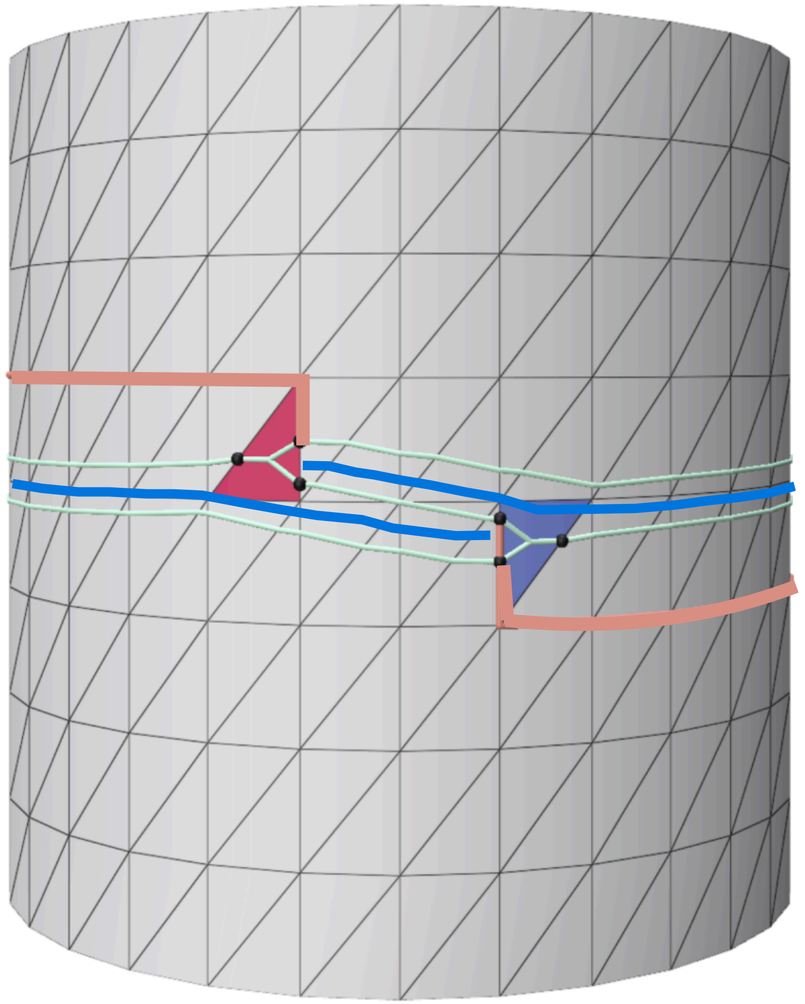
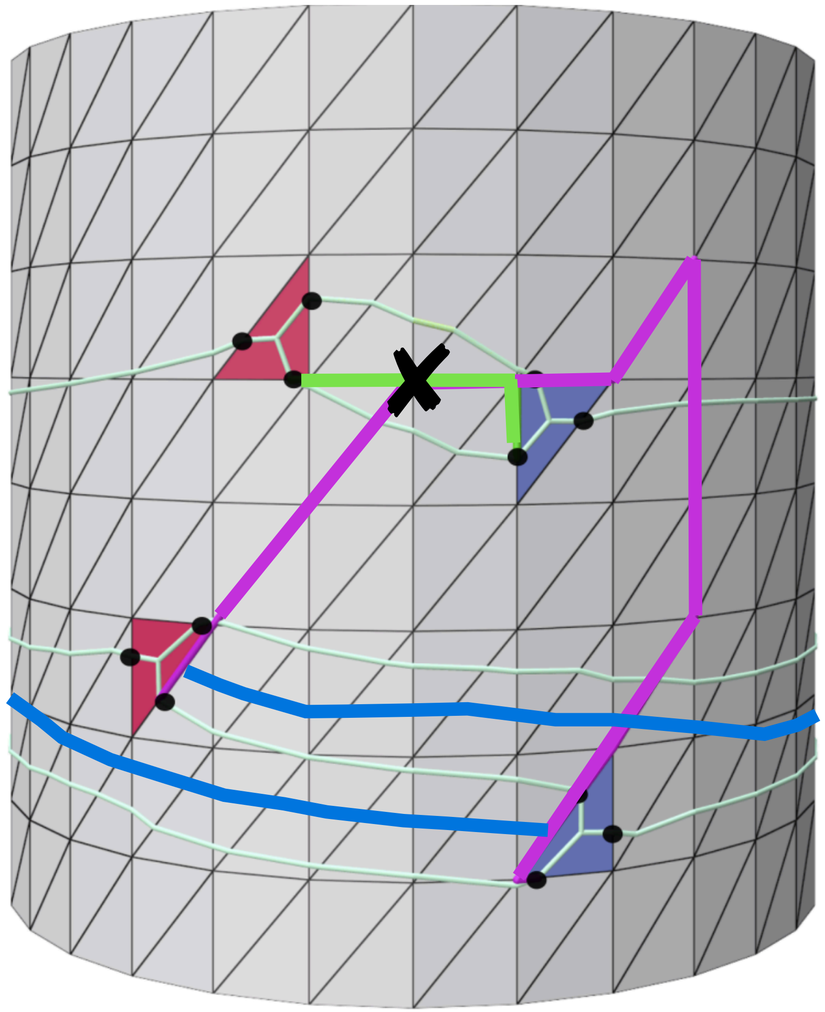
Automatic Singularity Matching



Automatic Singularity Matching



Automatic Singularity Matching

- Minimum weight matching LP

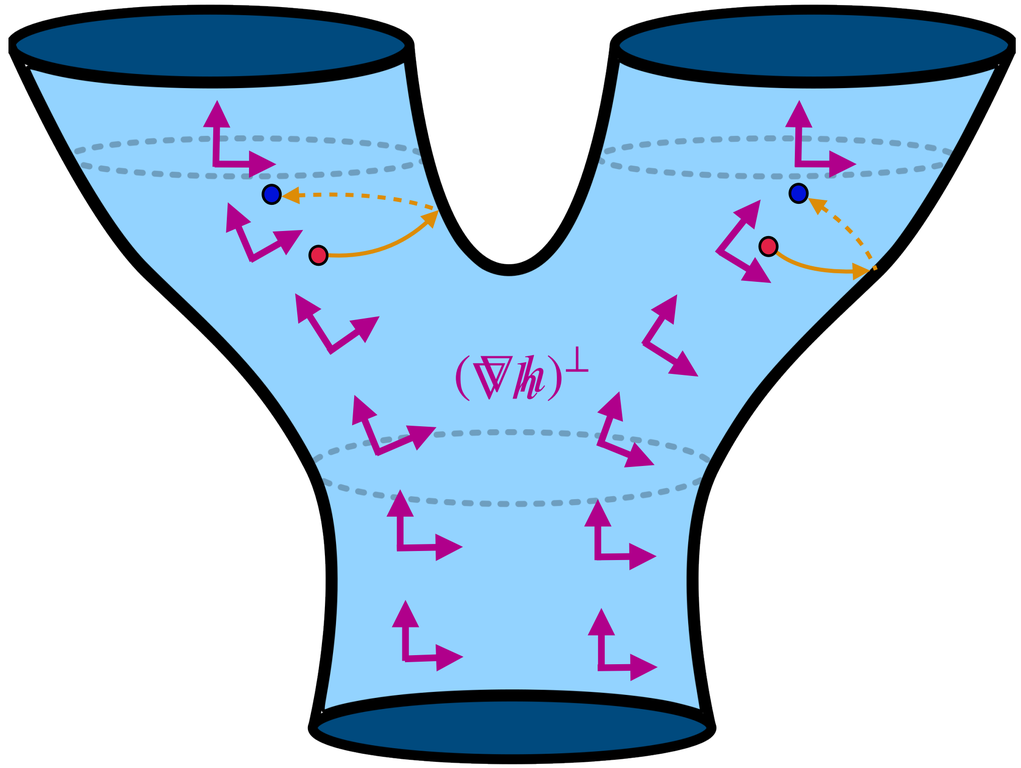
$$\min_{\mathbf{T}\in[0,1]^{s\times s}} \langle \mathbf{C}, \mathbf{T} \rangle$$

such that

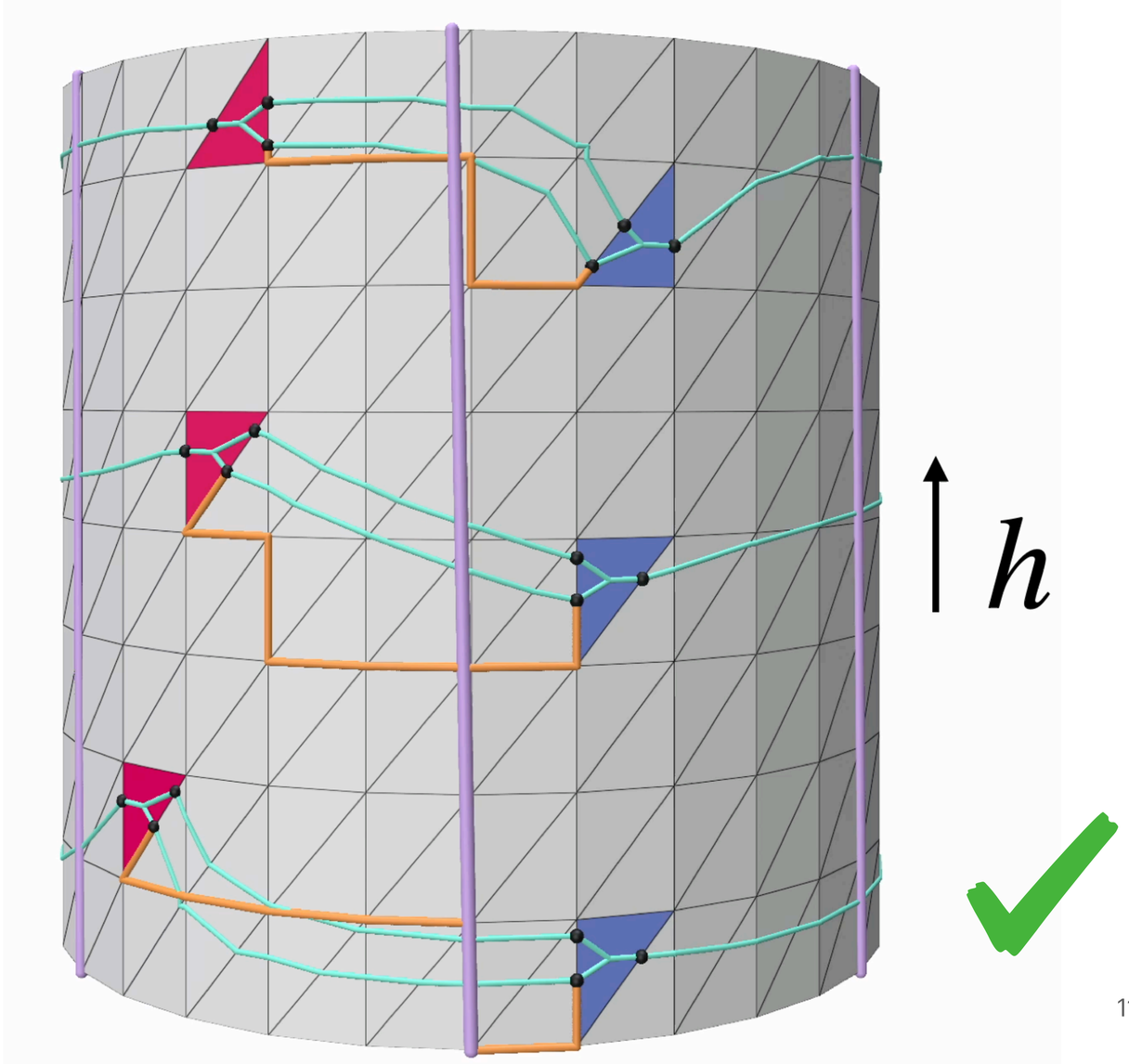
$$\mathbf{T}\mathbb{1} = \mathbb{1}$$

$$\mathbb{1}^T\mathbf{T} = \mathbb{1}^T$$

$C \in (R^+)^{s\times s}$, C_{ij} = cost of assigning singularity i to singularity j

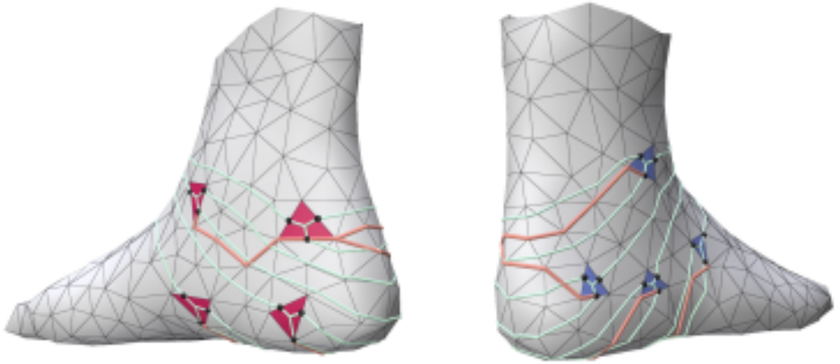


Automatic Singularity Matching - Result



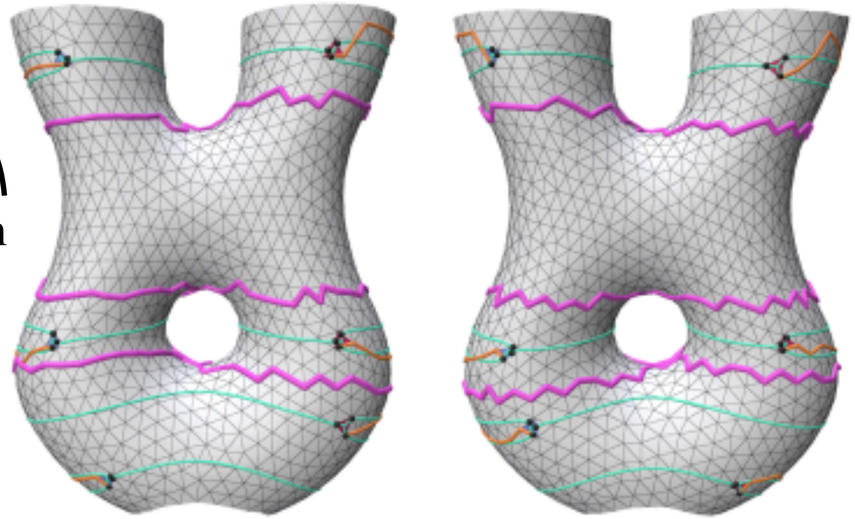
- Singularities on the same isoline matched ✓
- Path constraints flow from positive singularity to negative singularity ✓
- Non-intersecting path constraints ✓

Results



Change graph resolution while maintaining constraints

Models with genus



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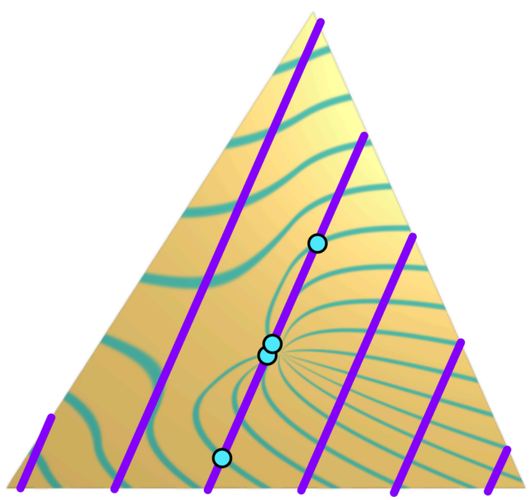
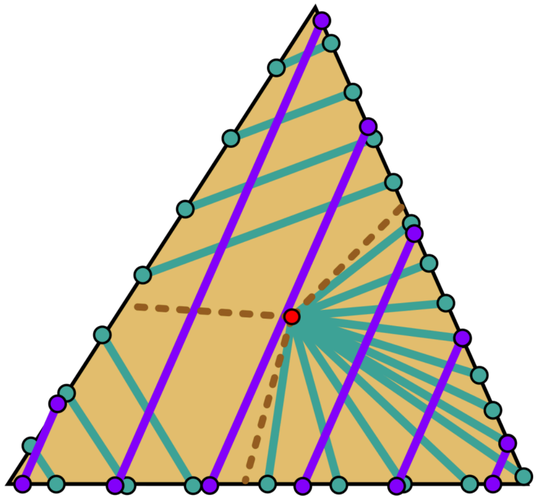
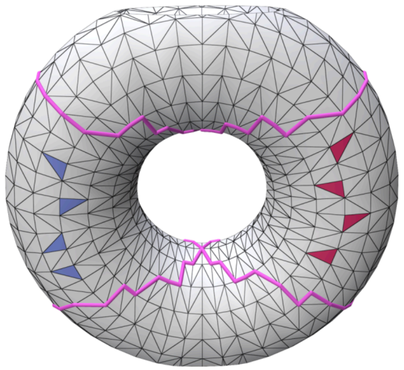
Summary

- Stripe patterns as foliations to achieve more precise control preventing helicing of **any** integral curve
- Automatic matching of singular triangles
- Effective Interpolant for robust stripe tracing
- Extension to models with genus via Morse decomposition







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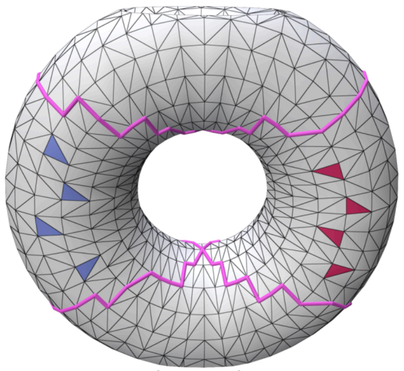

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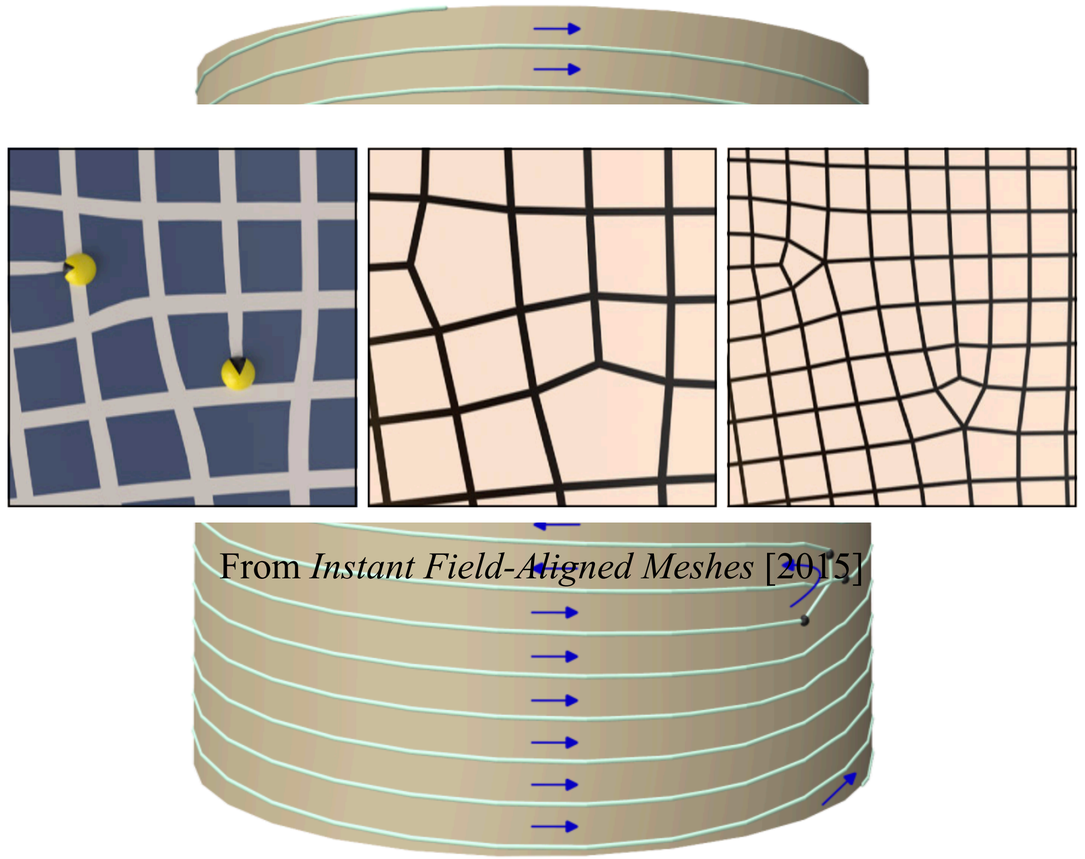
Future Work

- Stripe singularities
  - Generation?
  - Optimal Placement?
  - Position singularities in quad-dominant meshing
    - Curl quantization

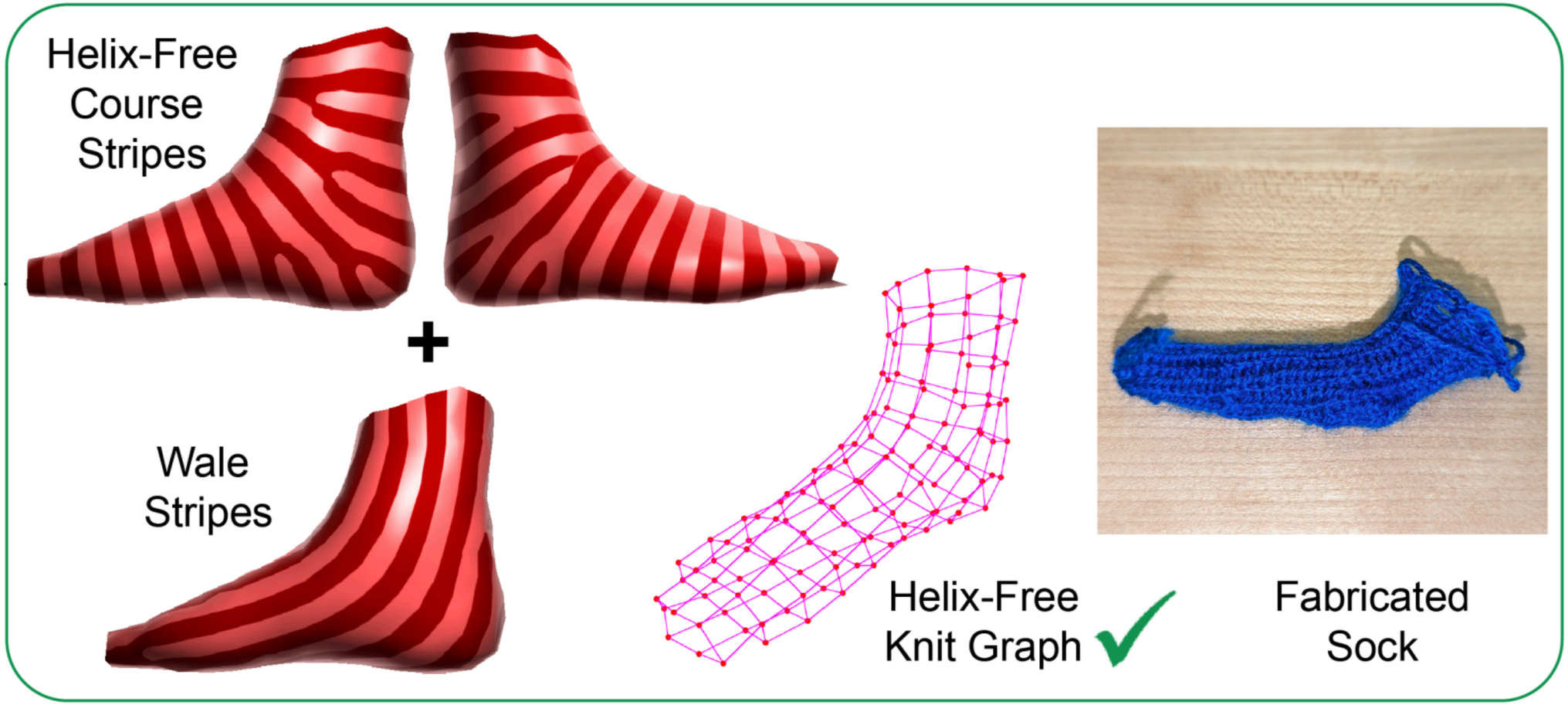
- Tracing-free pipeline
  - Tracing actually takes place as a helix



From *Instant Field-Aligned Meshes* [2015]

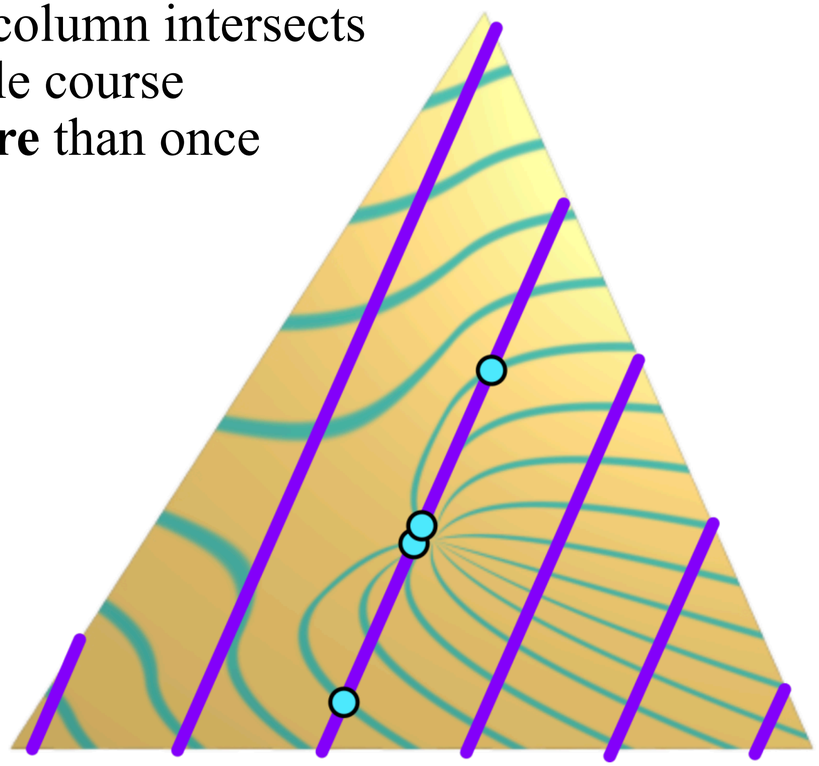
Thank you for your attention!
Come chat with us!
(Slides that follow are for clarifying questions)

Knit Graph Generation

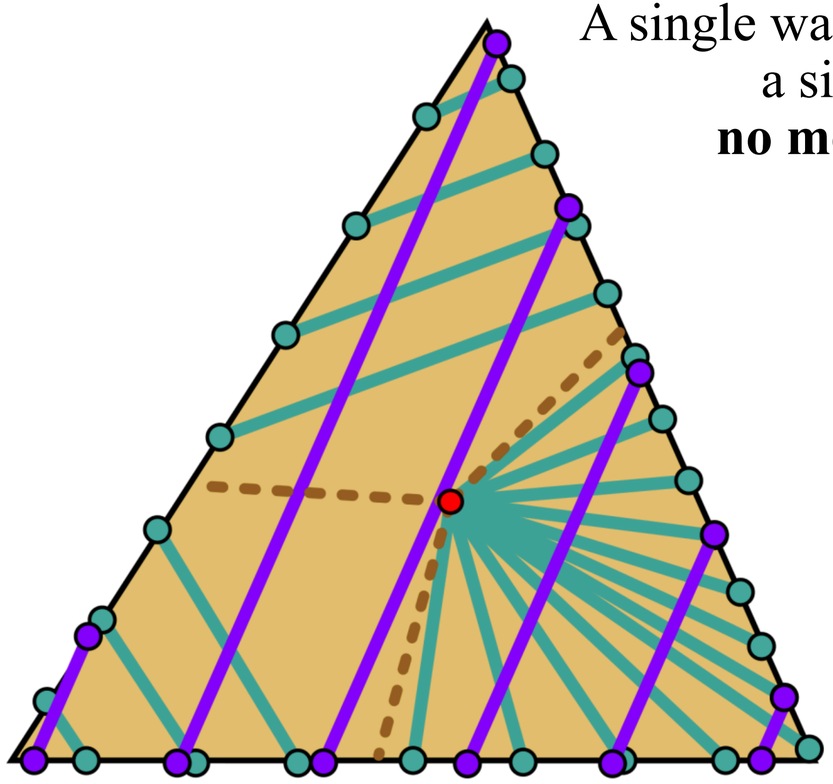


Effective Interpolant

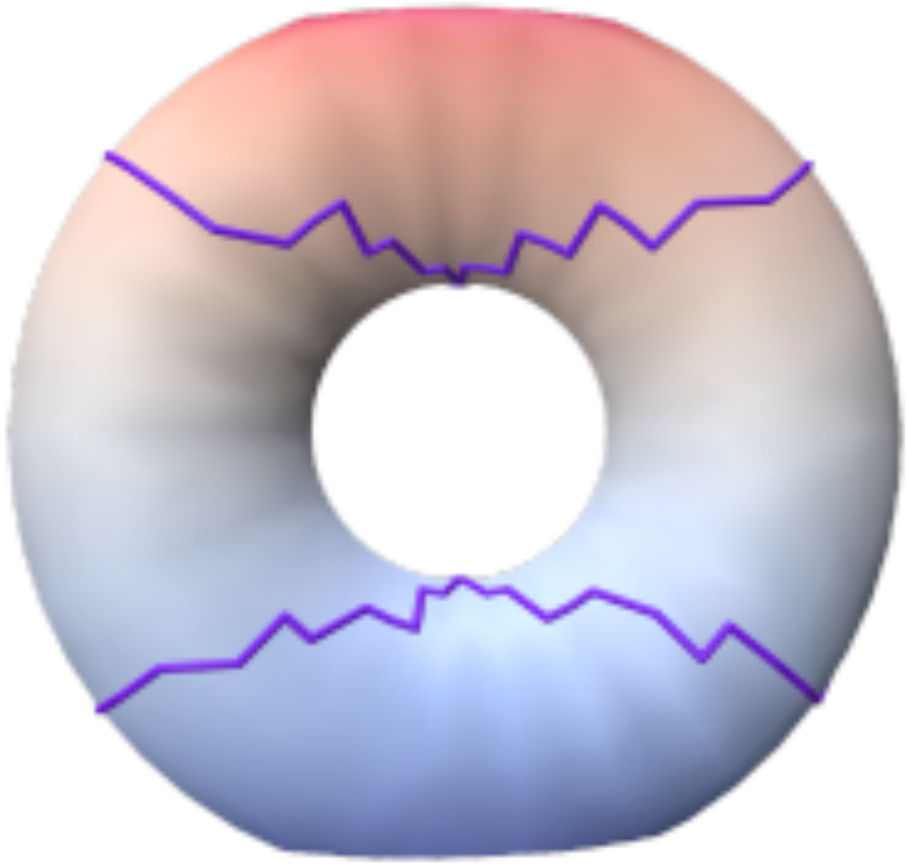
A single wale column intersects a single course stripe **more** than once



A single wale column intersects a single course **no more** than once

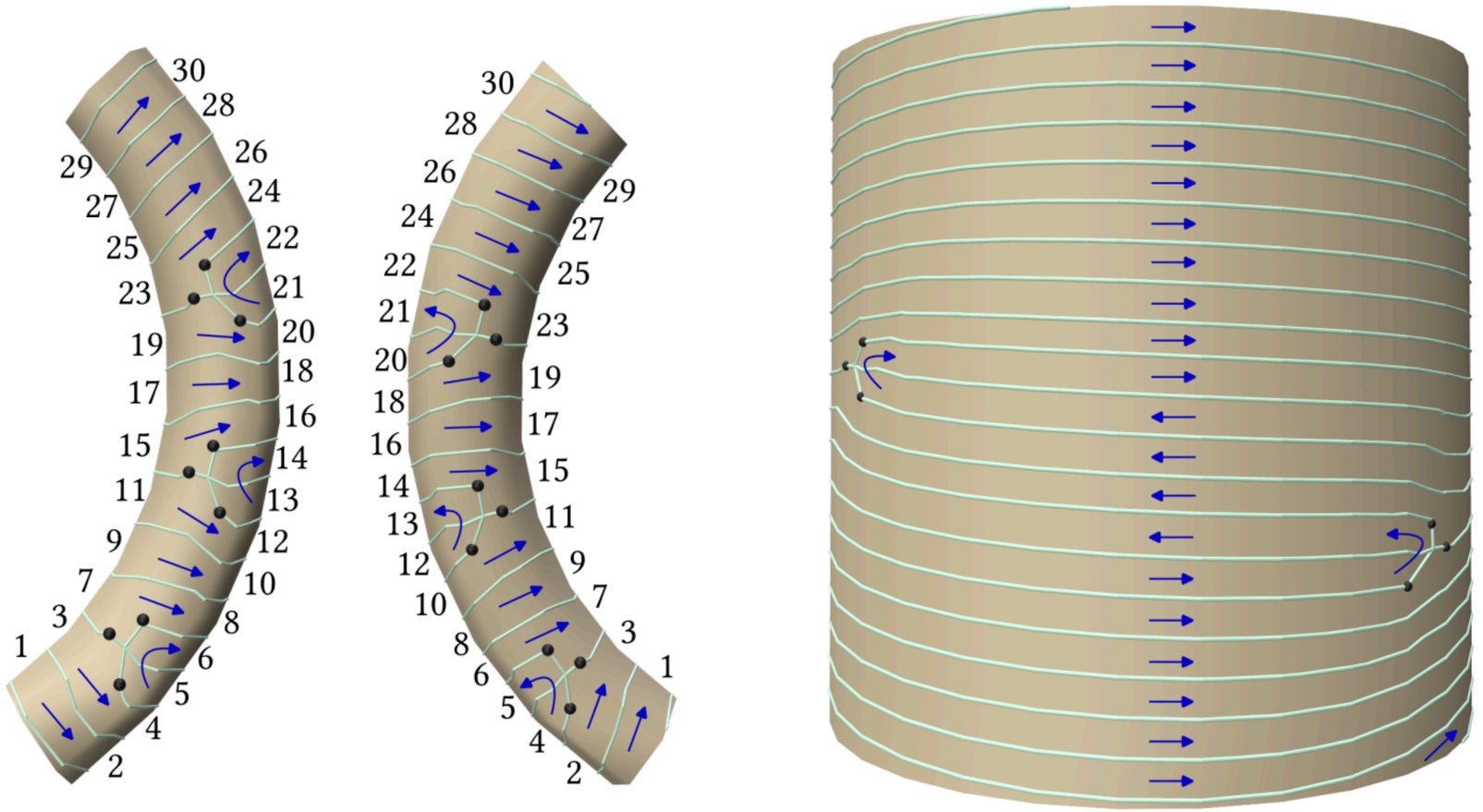


Morse decomposition for models with genus



Decompose surface *M* into cylinders by cutting along critical level sets (in a Morse sense) of the time function. Critical level sets of time function are values of time function at saddle vertices of *M*.

Tracing-free Pipeline



Tracing (yarn path) implied directly by foliation structure

1-form Optimization Problem

$$\min_{\sigma_c, \mathbf{k}^{\mathrm{hg}}} \quad ||W(\sigma_c - \omega_c)||^2$$

$$\text{subject to} \quad \sigma_c\big|_{\partial M} = 0,$$

$$d_1 \sigma_c = P\mathbf{k}$$

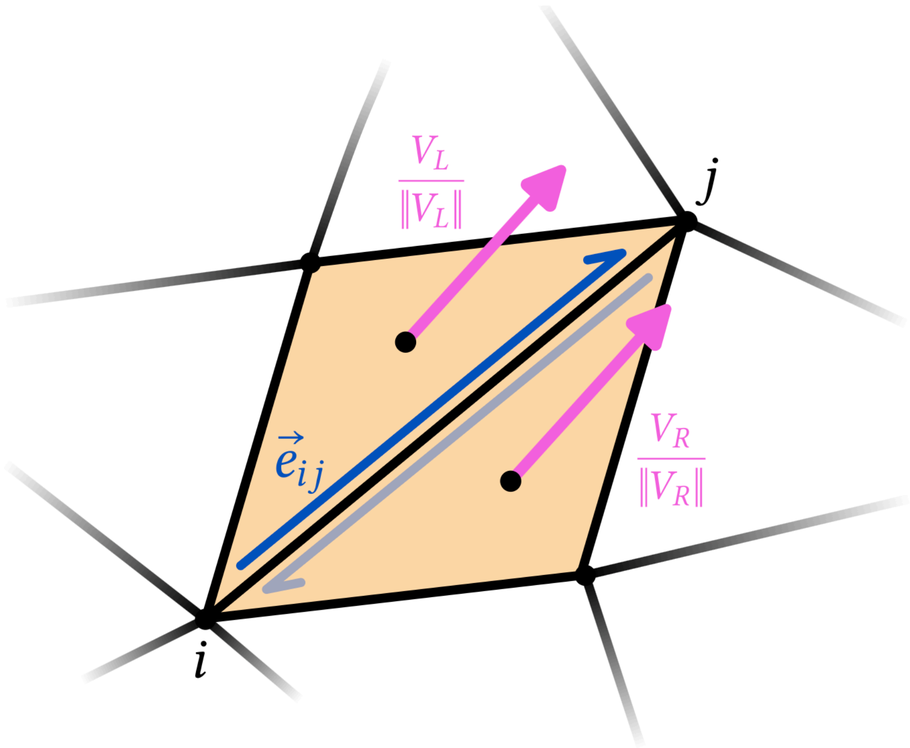
$$\mathbf{H}\sigma_c = P\mathbf{k}^{\mathrm{hg}}$$

$$\int_{\gamma_j^{\mathrm{ls}}} \sigma_c = 0, \quad 1 \le j \le N^{\mathrm{ls}}$$

H - Homology generators for models with genus

Only need **level-set constraints.** [Mitra et al. 2023] required level-set constraints, helix elimination constraints, stripe alignment constraints, stripe placement constraints!

Halfedge weights



$$w_{e_{ij}} = \left| C - \vec{e}_{ij} \cdot \frac{V}{\|V\|} \right|$$

where:

$$C = \max_{e_{ij}} \left\{ \vec{e}_{ij} \cdot \frac{V}{\|V\|} \right\}$$